

GEOLOGICAL NOTES

KELVIN AND THE AGE OF THE EARTH¹

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ABSTRACT

Kelvin's estimate of about 10^8 years for the age of the earth, based on the time required for an initially uniform and hot thermal structure to decay to the presently observed surface temperature gradient, is briefly reviewed. The later realization that the earth was at least several billion years old (4.5×10^9 years) posed a dilemma: How to reconcile thermal evolution with this great age? Radiogenic heat production alone is demonstrably insufficient to effect the reconciliation and, in the end, it is mantle convection, with its ability to exploit the entire internal heat of the earth, that provides the way out. The present usefulness of thermal structure as a measure of age (age now taken to mean local as opposed to global age) is reviewed for oceanic and continental regions. As for Kelvin, he was quite right in arguing that the earth's age was not only finite but measurable, measurable however by methods he did not anticipate.

INTRODUCTION

The true vastness of geologic time, and the overthrow of the Biblical chronology that would measure it in generations of man, is in my view the most profound of geology's many contributions. The subject is replete with legendary confrontation (see Burchfield, 1975): Uniformitarians such as Lyell, for whom time was without bound and the earth essentially unchanging vs. Kelvin, who was adamant that time was not only finite but measurable through the application of sound physical principles. Among these principles was heat conduction, which he used to determine the span of time required for the earth, starting from a hot initial state, to reach its present thermal structure. The great irony, and the source of many homilies, is the fact that he was unaware of radioactivity, which on the one hand acts as a long-lived heat source and on the other provides a separate measure of the earth's true age: 4.5 b.y. or about two orders of magnitude greater than that estimated by Kelvin.

The purpose of this paper is to restate Kelvin's (1863) problem for the thermal evolution of the earth in terms of having to recon-

cile the present thermal state of the earth with its great age. It will be seen that, despite conventional wisdom, the addition of a reasonable amount of heat producing radioactive elements does not itself suffice to effect such a reconciliation. Some other process must be operating, and for a time it was believed to be radiative transfer, which at the high temperatures of the earth's interior might allow more of the primordial heat to escape. The appeal to enhanced radiative transfer and *ad hoc* distributions of heat producing elements seems a classic case of necessity being the mother of invention (see MacDonald 1963 for one of the last great efforts to provide a theory of the thermal structure in which convection plays no part, serving also to illustrate the state of affairs exactly 100 years after the original work). The key process is in fact thermal convection, a process with which Kelvin was quite familiar in other contexts having used it in connection with the adiabatic thermal structure of the atmosphere.

KELVIN'S MODEL

In the early part of his classic paper on the secular cooling of the earth (1863), Kelvin states the problem quite succinctly: "The chief object of the present communication is to estimate from the general increase of temperature in the Earth downwards, the date of the first establishment of that *consistentior status*, which, according to Leibnitz's theory, is the initial date of all geologic his-

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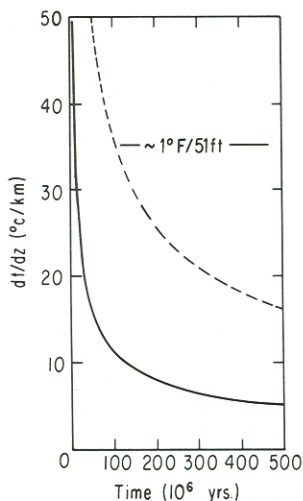


FIG. 1.—Temperature gradient at the surface as a function of time for a one-dimensional conduction model cooled from above and starting from an initially uniform temperature of 3888°C (dashed curve) and 1200°C (solid curve). As a measure of the age of the earth, Kelvin took the time required for the gradient to fall to 1°F/51 ft or about 35°C/km.

tory.” In mathematical terms the problem becomes that of solving

$$\frac{dT}{dt} = \kappa \frac{d^2T}{dz^2} \quad (1)$$

subject to a boundary condition of $T = 0^\circ\text{C}$ at the surface ($z = 0$) and starting from an initially uniform interior temperature of 3888°C (the value supposed by Kelvin to represent molten rock, modified to 1200°C in a later paper). For “the general increase of temperature in the Earth downwards” a value of 1°F/51 ft ($\sim 35^\circ\text{C}/\text{km}$) was used, based on measurements of underground temperatures near Edinburgh; these same measurements serving to estimate κ ($\sim 1.18 \times 10^{-2} \text{ cm}^2/\text{sec}$).

The evolution of the temperature gradient at the surface for the two assumed initial temperatures is shown in figure 1. The colder initial temperature gives an age estimate of about 20 m.y., which is the most quoted value, but in the original 1863 work Kelvin states “if we suppose the temperature of melting rock to be 7000°Fahr., we may suppose the consolidation to have taken place 98,000,000 years ago.”

We now know the age of the earth to be more nearly 4.5 b.y., and the question becomes how to modify Kelvin’s model so as to

keep the surface gradient from falling far below the observed gradient. It helps a bit (but not much really) to note that Kelvin’s estimate of the present average geothermal gradient at the surface is too large by about a factor of two. We might add a reasonable amount of radioactivity to the model, but what is reasonable? For this we need an estimate of the bulk composition of the earth, particularly the concentration of heat producing elements U, Th, and K. Bulk compositions when translated into global heat production rates fall in the range 1.5×10^{13} to 2.7×10^{13} watts (see McKenzie and Richter 1981, table 5), from which we can already anticipate trouble, since these production rates are significantly lower than the present heat flow out of the earth of 4.2×10^{13} watts (Sclater et al. 1980).

The Kelvin model, now including radioactive heating, takes the form

$$\frac{dT}{dt} = \kappa \frac{d^2T}{dz^2} + Q(z) \quad (2)$$

with the boundary condition $T = 0^\circ\text{C}$ at the surface. For the initial temperature we can now use the 1350°C adiabat, which corresponds to present estimates of the earth’s interior temperature (see Jeanloz and Richter 1979 for a recent estimate of the thermal structure of the mantle). $Q(z)$ is the volumetric heating rate as a function of depth.

The evolution of the surface temperature gradient shown in figure 2 assumes:

$$\begin{aligned} Q(z) &= 1.5 \times 10^{-6} & 0 < z < 10 \text{ km} \\ & \text{W/m}^3 & \text{(crust)} \\ Q(z) &= 1.5 \times 10^{-8} & z > 10 \text{ km} \\ & \text{W/m}^3 & \text{(mantle)} \end{aligned}$$

These heat production rates result in a total heat production of 2.25×10^{13} watts (toward the high end of the bulk composition models), with one-third of the heat-producing elements concentrated in the crust. As shown in figure 2, the calculated surface gradient falls below the world average gradient of about 20°C/km at about 10^8 yrs, regardless of the inclusion of radioactive heat generation. A few remarks are in order regarding the use of 20°C/km as the “observed” average surface temperature gradient. What is generally estimated is the average heat loss from the earth, which corresponds to about 80 mW/m². Using a rela-

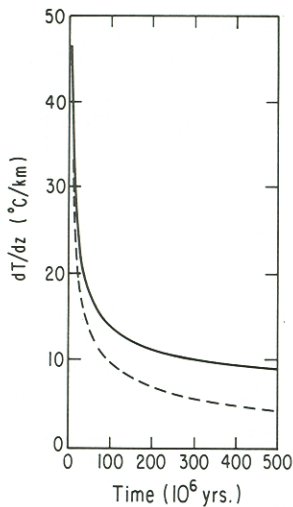


FIG. 2.—Temperature gradient at the surface as a function of time for a one-dimensional conduction model cooled from above, starting from an initially uniform temperature of 1350°C , and including radiogenic heat production (solid curve). The dashed curve is the equivalent model without heat production. The rate of heat production is $1.5 \times 10^{-6} \text{ W/m}^3$ in the uppermost 10 km and $1.5 \times 10^{-8} \text{ W/m}^3$ below, which represents the heat generation of a typical bulk composition for the earth. The inclusion of radiogenic heat makes little difference in terms of the age inferred from the time required for the surface gradient to reach its modern "observed" value of 20°C/km .

tively large value of $4 \text{ W/m}^{\circ}\text{C}$ for thermal conductivity, one obtains the 20°C/km from this average heat flux. This conductivity is typical for the mantle portion and somewhat larger than what would be appropriate for continental crust, but the use of a smaller conductivity would only raise the appropriate average surface temperature gradient.

The general conclusion is that, even if Kelvin had included a reasonable radiogenic heat production in his thermal evolution models, he would still have found grounds for arguing that the age of the earth was of the order of 10^8 years. The essential missing process is not really radiogenic heating at all but thermal convection, which allows the surface flux to exploit the entire internal heat of the earth as opposed to simply that of a shallow conductive boundary layer. Once one considers the earth as a whole a secular cooling of order $100^{\circ}\text{C}/10^9$ yrs is sufficient to maintain the present day rate of heat loss even in the absence of radiogenic heat generation.

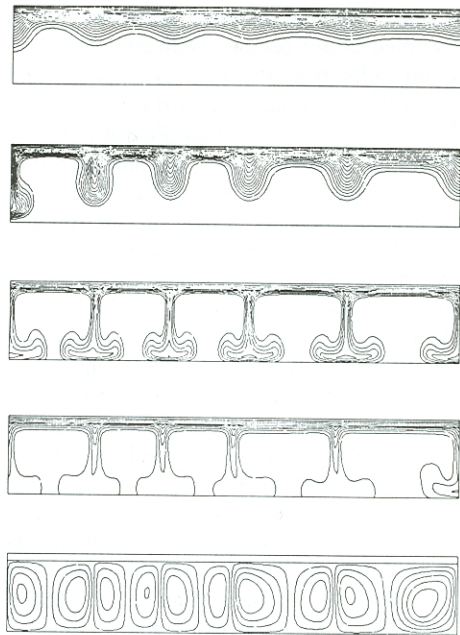


FIG. 3.—Isotherms at various times for a system consisting of a rigid lid (0.1 of the total depth) overlying a deformable layer. After an initial purely conductive stage, convective perturbations begin to appear (top panel). The subsequent evolution is one of amplification, culminating in fully developed convective regime under the conductive lid. The lowest panel shows the cellular streamlines of the fully developed flow.

The effect of thermal convection on the evolution of heat loss in a system cooled from above is illustrated in figures 3 and 4. The model consists of a rigid lid (representing the lithosphere) overlying a deformable layer. When cooled from above the system begins by cooling conductively until the thermal gradient begins to affect the deformable part,

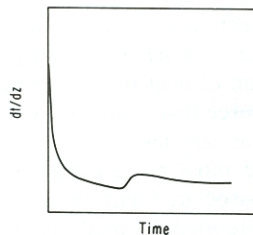


FIG. 4.—Evolution of the surface gradient for the model shown in figure 3. After any initial conductive cooling stage (identical in form to that of fig. 1), convection sets in and the gradient is increased and maintained at the higher level for a long period of time.

and cold, more dense fluid falls away from the cooled upper region. With time the convection cells become well established and transport heat to the base of the lid, thereby helping maintain the surface flux at a higher and more uniform level (fig. 4). The source of the heat being carried by the convection regime is a combination of heat production and secular cooling of the entire convective region, which for the earth would be the entire mantle and core.

There is one other point worth emphasizing when discussing Kelvin's use of thermal evolution to bound the age of the earth. Why should one assume that *local* thermal structure is related to *global* thermal evolution? The contemporary point of view is that local thermal structure depends on local age, i.e., the time since formation of the particular terrain or the time since it was thermally disturbed (reset) by major tectonism (see Sclater et al. 1980). It seems worthwhile to consider how Kelvin's model applies once the emphasis shifts to "local age."

OCEANIC MODELS

The key to understanding the near surface thermal structure of oceanic regions is plate tectonics. Hot material ascends adiabatically at mid-ocean ridges, becomes part of the plate, and subsequently cools conductively from above as it moves away from the spreading center. To a very good first approximation the appropriate model is the one used by Kelvin, except that now time is equated with age of the sea-floor. The simple idea is that the thermal structure under any part of the sea-floor is determined by taking a uniform high temperature ($\sim 1350^\circ\text{C}$) and cooling it conductively from above for a length of time equal to the local age of the sea floor. By using plate tectonics as the context for the model problem, we have implicitly accepted the convection of heat from the deeper interior as the source that supplies the initial high temperature at zero age.

One of the most remarkable geophysical discoveries associated with plate tectonics is that the simple model stated above does indeed account for the general dependence on age of both the heat flow and bathymetry of ocean basins (see Parsons and Sclater 1977). By recognizing that the relevant attribute is age, as opposed to distance (which depends

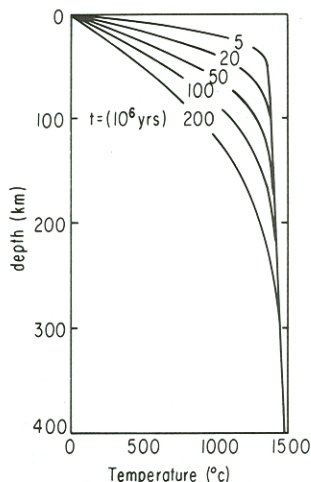


FIG. 5.—Temperature as a function of depth and time for a one-dimensional model starting from an initial adiabatic temperature $T(o,z) = 1350 + 0.3Z$ with Z in km. The thermal diffusivity is taken to be $10^{-6} \text{ m}^2/\text{sec}$, the density is $3.5 \times 10^3 \text{ kg/m}^3$ and the specific heat is $1.2 \times 10^3 \text{ J/kg } ^\circ\text{C}$. This evolution of temperature is consistent with the bathymetry of ocean basins at ages less than 80 m.y.

on both age and plate velocity), the heat flow and bathymetry of different oceans are found to be very much the same.

Figure 5 shows a "Kelvin" thermal evolution for an oceanic plate, which can be tested by geophysical data, the most diagnostic being bathymetry. As the initially hot plate cools, its density increases. Since there is little or no free-air gravity anomaly associated with mid-ocean ridges (i.e., isostasy prevails), this increase in density should result in decreasing topographic height (increasing ocean depth). If density is a linear function of temperature (i.e., characterized by a constant coefficient of thermal expansion) one predicts that the ocean should deepen in proportion to the square root of age. Figure 6, taken from Sclater et al. (1981), shows the relation between mean depth and the square root of age for data from the North Atlantic (slow spreading) and North Pacific (fast spreading). The important points are: (i) North Atlantic and Pacific are virtually identical when bathymetry is plotted against a function of age; (ii) over most of the ocean (age ≥ 80 m.y.) the depth varies in proportion to the square root of age, in agreement with simple "Kelvin" thermal evolution; (iii) at ages greater than

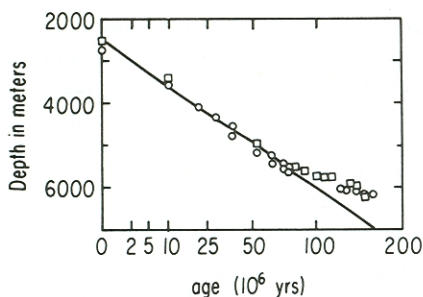


FIG. 6.—Mean depth of the North Atlantic (squares) and North Pacific (circles) vs. the square root of age, taken from Sclater et al. (1981). The solid straight line is the expected deepening from simple one-dimensional conductive cooling from above.

about 80 m.y. the depth begins to depart from the simple square root of age rule in the sense of being shallower than it ought to be for purely conductive cooling. This figure summarizes the best evidence both for and against the Kelvin model: it works but it eventually breaks down! The breakdown is most likely associated with a convective slowdown in cooling of the sort illustrated earlier in figures 3 and 4. In general the thermal structure is a good and useful measure of age, but by age we now mean something quite different than the age of the earth.

CONTINENTAL MODELS

Because of the high concentration of heat-producing elements in the continental crust any discussion of the thermal regime and its evolution must first deal with the question of how much of the observed heat flux is due to

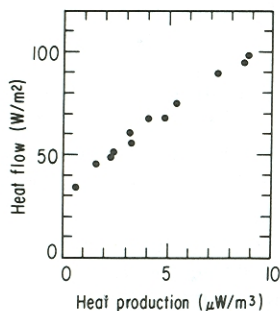


FIG. 7.—Heat flow vs. heat production for the eastern United States taken from Roy et al. 1968. The data fall along a straight line which, when extrapolated to zero heat production, gives a measure of the reduced heat flow which represents the heat coming from below the near surface radiogenic layer.

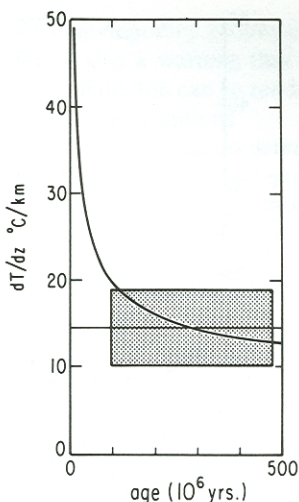


FIG. 8.—Evolution of the surface gradient for a one-dimensional conduction model cooled from above and including near surface heat generation consistent with the observations shown in figure 7. The initial temperature assumed is the 1350°C adiabat and $Q(z) = (3.2 \times 10^{-6} e^{-z/7.5} + 1.5 \times 10^{-8})$ W/m³ where z is measured downwards from the surface in km. The heat production parameters are taken from table 1 in Jaupart et al. (1981). The shaded box represents the "observed" surface gradient $\pm 1\sigma$ and age since the last major tectonic events (Arcadian orogeny to White Mountain Magma Series). Given the relatively young tectonic age of this province, a conduction model with surface radioactivity can account for all the observations.

near-surface radiogenic heating. The approach most commonly used is that of Birch et al. (1968), who noted a linear relationship between heat flow and surface radiogenic heat production (see fig. 7), which when extrapolated to zero heat production gives an estimate of the heat flux from below the radiogenic layer. In terms of a Kelvin model one might ask whether the reduced heat flow can be explained by simple conductive cooling.

From the linear relation in figure 7 one estimates a reduced heat flow of 34 mW/m² and a radiogenic layer in which the heat production decreases downward exponentially, with a length scale of 7.5 km, from a maximum value of 3.2 μ W/m³ (Roy et al. 1968). The evolution of the surface temperature gradient corresponding to this heat production profile is shown in figure 8. When compared to the New England heat flow data, assuming that

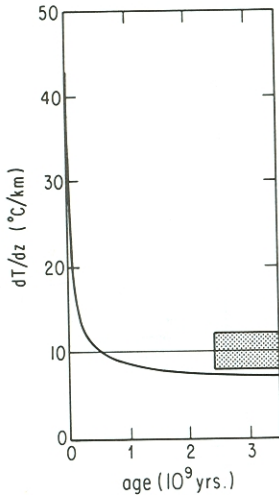


FIG. 9.—Same as figure 8 but for the Western Australian shield with a tectonic age in the range 2.4 to 3.4 b.y. The shaded box represents the surface gradient associated with an average heat flow of $39 \pm 8 \text{ mW/m}^2$. For heat production $Q(z) = (2.9 \times 10^{-6} e^{-z/4.5} + 1.5 \times 10^{-8}) \text{ W/m}^3$ is used (table 1, Jaupart et al. 1981). Because of the great age of this province there is a suggestion that conduction plus surface heat generation alone results in too small a surface temperature gradient to account for the actual observations.

the appropriate "age" is that since the last major tectonic event (which would have reset the thermal structure), it is seen that a conductive model with radioactivity can indeed account for the observations. It is only when one considers a very old stable craton that one finds a suggestion that something besides conduction and heat generation is operating. Figure 9 is similar to figure 8, but for the Western Australian shield with a tectonic age greater than 2.5 b.y. The calculated temperature gradient now falls below the gradient one infers from the heat flow data, suggesting a convective contribution of heat to the base of the continental lithosphere, possibly the same process required to explain the bathymetry at old ages in the oceans.

In contrast to the situation in ocean basins, the thermal structure of continental regions is a very poor measure of age, even when radiogenic heat production is taken into account.

THE CONTEMPORARY PROBLEM

A more complete discussion of thermal evolution must include secular cooling, con-

vective transport of heat, conduction, and radiogenic heat production. For simplicity let us consider the problem in cartesian coordinates and ignore effects arising from the compressibility of the medium. The energy equation becomes

$$\rho C_p \left\{ \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right\} = K \nabla^2 T + Q \quad (3)$$

where ρ is density, C_p is the specific heat at constant pressure, \mathbf{u} is the convective velocity field, K is the thermal conductivity and as before Q is the rate of radiogenic heat production. The complete problem requires extra equations; those of mass conservation and momentum, which specify the dynamics from which in principle the flow field can be calculated. In practice the complete problem proves intractable because of the extreme variation in material properties with temperature. This difficulty is most obvious in specifying the ease of deformation which ranges from virtually undeformable at the cold surface (accounting for the rigid plates of plate tectonics) to sufficiently deformable in the interior so that heat transport is totally dominated by the convective term. Because of the intractability of the complete problem, what is typically done is to integrate equation (3) over the volume of the earth yielding an equation for the evolution of the planetary mean temperature (see McKenzie and Weiss 1975; also Richter 1984)

$$\frac{\partial}{\partial t} \int \rho C_p T dv = -F + H \quad (4)$$

where F is the total heat loss from the earth and H the total rate of radiogenic heat production. Even if the rate of heat generation is specified by an estimate of the bulk composition, an extra relation between heat flux and average temperature is required to close the problem. The relation between flux and temperature has to reflect the role of both conduction in boundary layers and convection in the interior, and is typically obtained from laboratory or numerical experiments with simpler convecting systems. Let us assume that a relationship

$$F = f(T, \dots) \quad (5)$$

exists and specifies the flux at the surface of the earth given the average temperature and other properties of the system. For a given function f and bulk composition model, equations (4) and (5) can be combined into a single

equation for the evolution of the planetary mean temperature.

The contemporary problem is to find convective models for the earth which yield a function f such that when (4) is integrated for 4.5 b.y. it results in a rate of secular cooling equal to the difference between the observed rate of heat loss ($-F$) and heat generation (H) associated with estimates of the bulk composition of the earth. Thus the thermal evolution problem becomes in effect an important constraint on the style and heat transfer efficiency of models for mantle convection.

SUMMARY

In the broadest sense Kelvin was quite right about two important aspects of the thermal evolution and age of the earth. The age of the earth is finite and measurable, and even though Kelvin's particular method was flawed, the goal he posed was one of crucial importance. The second point has to do with his treating the earth as if it were a hand-held object and just as subject to the laws of physics as any other hand-held object. It represents a very modern attitude that is at

the heart of contemporary geophysical modeling. There is also a warning that even the most elegant calculation can be rendered misleading by faulty assumptions.

The original problem was to determine the age of the earth from a thermal model. The subsequent discovery of radioactivity completely changed the subject, not so much because of the associated heat production, but because it gave a separate and different measure of the age of the earth. The modern problem is almost the reverse of the original: given the great age of the earth one seeks a thermal and dynamical model that can account for the present thermal state. Such a model must take account of not only radiogenic heat but also the role of mantle convection as a means of exploiting the entire heat content of the earth.

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